Math 146C - Ordinary and Partial Differential Equations III
Quiz 2
April 14, 2011

Name: Key

1	2	Total
/7	/3	/10

Problem 1. *(7 points) Assuming that*

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right), \tag{1}$$

show formally (meaning don't worry about convergence of the series) that

$$\frac{1}{L} \int_{-L}^{L} [f(x)]^2 \, dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2).$$

Hint: Multiply equation (1) by $f(x)$ and integrate from $-L$ to L.

Problem 2. *(3 points) Prove or disprove the following statement:*
Let $f : \mathbb{R} \to \mathbb{R}$ be an odd function. Then $f(0) = 0$.

① Multiply by $f(x)$ and integrate from $-L$ to L:

$$\int_{-L}^{L} [f(x)]^2 dx = \frac{a_0}{2}\int_{-L}^{L} f(x)\, dx + \sum_{n=1}^{\infty}\left(a_n \int_{-L}^{L} f(x) \cos\frac{n\pi x}{L} dx + b_n \int_{-L}^{L} f(x) \sin\frac{n\pi x}{L} dx\right)$$

$$= \frac{a_0}{2}(La_0) + \sum_{n=1}^{\infty}\left(a_n(La_n) + b_n(Lb_n)\right)$$

$$= L\left(\frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)\right)$$

$$\Rightarrow \frac{1}{L}\int_{-L}^{L}[f(x)]^2 dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

② Odd function: $f(-x) = -f(x) \Rightarrow f(0) = f(-0) = -f(0)$

$\Rightarrow 2f(0) = 0 \Rightarrow f(0) = 0.$